

2 SD codes

Self-dual codes invariant under permutation groups

Let G be a prescribed permutation group. We discuss the question of existence of self-dual codes over a field $F = F_q$, $q = p^l$, p a prime, admitting G , that is, codes which are G -invariant. This depends on the structure of G and its representations as well as the base field F_q .

We examine conditions which are necessary and sufficient for the existence of such codes. Representation theoretic as well as group theoretic methods are used. For illustration purposes in the binary case we discuss the existence of self-dual binary codes of length n which are invariant under the symmetric groups S_n and the alternating groups A_n , $n \geq 4$ and show that such codes do not exist.

Further, we shall give some examples for the sporadic simple and almost simple groups of degree ≤ 2000 , and search for G -invariant self-dual codes of various lengths and will discuss an attempt to classify such codes where computations were possible, or theoretical methods permitted. Most of the material discussed here could be found in [1, 2].

One of the questions of current interest in coding theory is the following: given a finite non-solvable permutation group G acting transitively on a set Ω , under what conditions on G are self-dual codes invariant under G existent or nonexistent? In this lecture, we follow the ideas discussed in [3] and examine this problem under the hypothesis that the group G is an imprimitive rank 3 permutation group. We will give a brief overview of the problem and indicate that if G is an imprimitive rank 3 permutation group acting transitively on the coordinate positions of a self-dual binary code C then G is one of M_{11} of degree 22; $\text{Aut}(M_{12})$ of degree 24; $\text{PSL}(2, q)$ of degree $2(q+1)$ for $q \equiv 1 \pmod{4}$; $\text{PSL}(m, q)$ of degree $2 \times \frac{q^m-1}{q-1}$ for $m \geq 3$ odd and q an odd prime; $\text{PSL}(m, q)$ of degree $2 \times \frac{q^m-1}{q-1}$ for $m \geq 4$ even and q an odd prime, and $\text{PSL}(3, 2)$ of degree 14.

When combined with a result on the classification of binary self-dual codes invariant under primitive rank 3 permutation groups of almost simple type this yields a result on the non-existence of extremal binary self-dual codes invariant under quasiprimitive rank 3 permutation groups of almost simple type.

References

- [1] Tendai. M. Mudziiri Shumba. On the existence of self-dual codes invariant under permutation groups. Master's thesis, University of KwaZulu-Natal, 2015.
- [2] B G Rodrigues and T MM Shumba. Some remarks on self-dual codes invariant under almost simple permutation groups. In Groups St Andrews 2017 in Birmingham, 5th-13th August 2017: London Math. Soc. Lecture Notes, Vol 455 (2019). pages 469 – 485.
- [3] B G Rodrigues, On the classification of binary self-dual codes admitting imprimitive rank 3 permutation groups. Appl. Algebra Eng. Commun. Comput. **32** (2) (2021), 113–134.